REPETITIVE GROUP SAMPLING PLAN UNDER GAMMA-POISSON DISTRIBUTION

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ABSTRACT: *This paper proposes a Bayesian repetitive group sampling plan for the application of attribute quality characteristics under the conditions of gamma-Poisson distribution. The optimal design parameters of the proposed plan such as the sample size and the acceptance numbers are determined based on two points on operating characteristics curve approach and by minimizing the average sample number since any sampling plan with minimum average sample number is always preferred. Extensive tables are also developed for easy selection of the optimal parameters of the proposed plan for selected combinations of two quality levels and the results are explained with examples. The proposed plan can be applied when the lifetime of the product follows the Gamma-Poisson distribution.*

KEY WORDS: Repetitive group sampling; gamma-Poisson distribution; risks

1. INTRODUCTION

In recent years, product quality has become one of the most important aspects that differentiate different commodities in a global business market. To ensure the quality of the products, there are two major techniques available in the statistical quality control literature. They are termed as statistical process control and statistical product control. Acceptance sampling is one of the major areas of statistical quality control which comes under statistical product control. Inspection of raw materials or finished products using acceptance sampling is one aspect of quality assurance. It is well known that the acceptance sampling plans are used to reduce the cost of inspection. The acceptance sampling is also useful in situations where testing is destructive, the cost of 100% inspection is extremely high, the time taken would be too long, the inspection error is too high and the product liability risks are serious.

In the concept of acceptance sampling, three major areas of sampling are available, one is attributing sampling, the second one is variables sampling and the last one is mixed sampling. Attributes sampling constitutes one of the vital areas of acceptance sampling. There are several sampling plans available in the literature for the inspection attribute quality characteristics. Some of them are called conventional sampling plans and some of them are called special purpose plans. Single sampling plan, double sampling plan and sequential sampling plan are called conventional sampling plans. Some of the special purpose plans are chain sampling plan, multiple sampling plan, repetitive group sampling (RGS) plan, etc. Single sampling plan is one of the simplest attributes sampling plans which involves two parameters namely the sample size n and the acceptance number c. The double sampling plan can be used to minimize the producer's risk. In double sampling if the results of the first sample are not definitive in leading to acceptance or rejection of a lot, a second sample is taken which then leads to a decision on the disposition of the lot. This approach makes sense not only as a result of experience, but also the mathematical properties of the procedure. Various special purpose plans have been developed to serve for certain special purposes.

In all the attributes sampling plans, the basic assumption is that the lot or process fraction defective is constant, which indicates that the production process is stable. However, in practical situations, the lots formed from a process will have quality variations which are due to random fluctuations. These variations are classified into two, one is within-lot variation and the other is between-lot variation. When the second type variation is more than the first type of variation, the fraction non-conforming items in the lots will vary continuously. In such cases, the decision on the submitted lots should be made with the consideration of the second type of variation. so that the conventional sampling plans cannot be applied. Instead, sampling plans based on Bayesian methodology can be applied with the prior knowledge on the process variation in making a decision on the disposition of the lot. For further details about the prior and posterior distributions of the lot fraction non-conforming, one can refer Guild and Raka [1], Hald [2], Case and Keats [3] and Dyer and Pierce [4].

In the Bayesian theory, it is found that the gamma distribution is a natural conjugate prior for the sampling from a Poisson distribution. When the sample items are drawn randomly from a process, the number of defects (or nonconformities) in the sample is distributed according to Poisson law, and the gamma distribution is the conjugate prior to the average number of non-conformities per item. Under these situations, Hald [2] derived operating characteristic (OC) function of the single sampling plan based on gamma-Poisson distribution. Vijayaraghavan et al. [5] developed the tables for the selection of parameters of single sampling plan using the gamma-Poisson distribution. Balamurali *et al.* [6], have developed double sampling under the conditions of applications of gamma-Poisson model. More details can be seen in Aslam et al. [21]. It is to be pointed out that, in the literature, there is no repetitive group sampling plan is available for gamma-Poisson situation. So this paper attempts to develop a Bayesian RGS plan under gamma-Poisson model. The optimal parameters of the proposed plan can be determined plan for specified requirements under the conditions of gamma prior and Poisson distribution. The rest of the paper is set as: a brief introduction about the RGS plan under gamma-Poisson distribution is given in Section 2. The design of the proposed plan is given in Section 3. Comparison of the proposed plan with the existing plans will be made and discussed in Section 4. Some concluding remarks are given in the last section.

2. RGS PLAN UNDER GAMMA-POISSON DISTRIBUTION

The concept of repetitive group sampling plan was developed by Sherman [7] for the application of attribute quality characteristics. Through this plan, acceptance or rejection of the lot is based on the repeated sample results of the same lot. The operation of this plan is similar to that of the sequential sampling plan. According to Sherman, the RGS plan will give the minimum sample size with desired protection. Further, RGS plan is not nearly as efficient as the sequential sampling plan, but it is always more efficient than the single sampling plan. Several authors have investigated the RGS plan under various situations. Procedures and tables for the selection of the parameters of RGS plan have been given by Soundararajan and Ramaswamy [8] and Singh et al. [9]. Balamurali and Jun [10] developed RGS plan for the application of measurable characteristics under normal distribution. Jun et al. [11] developed variables RGS plan under failure censored reliability tests for Weibull distribution. Aslam et al. [12] studied variables RGS plan of Balamurali and Jun [10] with process loss consideration. More details about the applications of RGS can be seen in Aslam et al. [13]. According to Sherman [7], the operating procedure of the RGS plan is as follows.

Step (i) :Select a random sample of size n from the lot of size N and observe the number of nonconforming items in the sample, say d.

Step (ii): If $d \le c_1$, then accept the lot. If $d > c_2$, then reject the lot, If $c_1 < d \le c_2$,

Then repeat the steps (i) and (ii) until a decision is made on the lot.

Thus the RGS plan is completely specified by the parameters n, the sample size and c_1 and c_2, the acceptance numbers. If $c_1 = c_2 = c$ (say), then the RGS plan will converge to the single sampling plan.

Gamma-Poisson distribution, which is also known as the negative binomial distribution, is one of the most widely used models in various areas of statistics such as, food industry, accident analysis etc. (refer Lord et al. [15], Lord [16] and Toft et al. [17]). This model is also used in the acceptance sampling plans. Vijayaraghavan et al. [18] analyzed the properties of OC curve of the acceptance sampling plans based on gamma-Poisson distribution. Vijayaraghavan et al. [5] developed the selection of single sampling plan using Gamma-Poisson distribution. When the production process produces output in a continuous stream and observed number of defects in the sample drawn from this process is distributed as Poisson with parameter np, where n is the sample size and p is the average number of defects per unit (see Hald [2]). According to Schilling [19], the Poisson distribution is an appropriate model for the number of non-conforming items in the sample when the ratio of a sample size to the population size (n/N) is less than 10%, n is large and $p < 0.10$ is small such that np<5. According to Sherman [7], the probability of acceptance of the RGS plan is given by

$$P_a(p) = \frac{P_a}{P_a + P_r} \qquad (1)$$

Where P_a and P_r are the probability of acceptance and rejection of a lot respectively, when the fraction nonconforming, p. That is., $P_a = P(d \le c_1/p)$ and $P_r = P(d > c_2/p)$

where c_1 and c_2 are the acceptance numbers of the RGS plan. Under the conditions of applications of Poisson probability model, the above probabilities are given by

$$P_a = \sum_{d=0}^{c_1} \frac{e^{-np}(np)^d}{d!} \quad \text{and}$$

$$P_r = 1 - \sum_{d=0}^{c_2} \frac{e^{-np}(np)^d}{d!}$$

Hence the probability of acceptance of the RGS plan under Poisson model is given by

$$P_a(p) = \frac{\sum_{d=0}^{c_1} \frac{e^{-np}(np)^d}{d!}}{1 - \sum_{d=0}^{c_2} \frac{e^{-np}(np)^d}{d!} + \sum_{d=0}^{c_1} \frac{e^{-np}(np)^d}{d!}}$$

When p varies from lot-to-lot at random and is distributed as gamma distribution which is the natural conjugate prior for sampling from the Poisson distribution, the density function of prior distribution of p is given by

$$f(p/a,m) = \frac{e^{-ap} a^m p^{m-1}}{\Gamma m}, \quad 0 \le p < \infty, \; a, m > 0 \qquad (2)$$

where a is the scale parameter and m is the shape parameter. If $E(p) = \bar{p}$ is given then the scale parameter is obtained by $a = m/\bar{p}$. Here m is either specified or estimated from the prior information about the production process. The posterior distribution of the number of nonconformities is reduced to the gamma-Poisson distribution. When the production is unstable, both the number of nonconforming items in the sample, d and the average number of defects p are independently distributed. So, according to Hald [2], the sampling distribution of d, under the conditions that the process average $\bar{p} < 0.1, \frac{\bar{p}}{m} < 0.2$ is given by

$$P(d; n\bar{p}, m) = \frac{(m+d-1)!}{d!(m-1)!} \left(\frac{n\bar{p}}{m+n\bar{p}}\right)^d \left(\frac{m}{m+n\bar{p}}\right)^m,$$

$$d = 0,1,2,..... \qquad (3)$$

Based on this, the probability of acceptance of the RGS plan under gamma-Poisson model is given by

$$P_a(\bar{p}) = \frac{\sum_{d=0}^{c_1} \frac{(m+d-1)!}{d!(m-1)!}\left(\frac{n\bar{p}}{m+n\bar{p}}\right)^d \left(\frac{m}{m+n\bar{p}}\right)^m}{1-\sum_{d=0}^{c_2} \frac{(m+d-1)!}{d!(m-1)!}\left(\frac{n\bar{p}}{m+n\bar{p}}\right)^d \left(\frac{m}{m+n\bar{p}}\right)^m + \sum_{d=0}^{c_1} \frac{(m+d-1)!}{d!(m-1)!}\left(\frac{n\bar{p}}{m+n\bar{p}}\right)^d \left(\frac{m}{m+n\bar{p}}\right)^m}$$

(4)

3. DESIGNING OF RGS PLAN UNDER GAMMA-POISSON MODEL

In this paper, we use two points on the OC curve approach to design the RGS plan. Any sampling plan can be designed which should satisfy the producer's and consumer's risks with a minimum average sample number. The optimal plan parameters are determined to satisfying the following inequalities.

$$P_a(p_1) \geq 1-\alpha \quad \text{and} \quad P_a(p_2) \leq \beta$$

(5)

Here p_1 is the quality level corresponding to the producer's risk, which is called acceptable quality level (AQL). On the other hand, p_2 is the quality level corresponding to the consumer's risk which is also called limiting quality level (LQL). It is important to note that there may exist multiple solutions as there are only two equations with three unknown parameters. So we may determine these parameters to minimize the average sample number (ASN) at LQL, where ASN is defined as the expected number of sampled units per lot used for making decisions. The ASN of the RGS plan is given by

$$ASN(p) = \frac{n}{1-P_a+P_r} \qquad (6)$$

Where P_a and P_r are the probability of acceptance and rejection a lot respectively, under the gamma-Poisson model, which are given by

$$P_a = \sum_{d=0}^{c_1} \frac{(m+d-1)!}{d!(m-1)!}\left(\frac{n\bar{p}}{m+n\bar{p}}\right)^d \left(\frac{m}{m+n\bar{p}}\right)^m$$

$$P_r = 1-\sum_{d=0}^{c_2} \frac{(m+d-1)!}{d!(m-1)!}\left(\frac{n\bar{p}}{m+n\bar{p}}\right)^d \left(\frac{m}{m+n\bar{p}}\right)^m$$

Hence to develop tables for designing an optimal RGS plan, we use the following nonlinear programming problem.

Minimize $ASN(p_2)$

Subject to $P_a(p_1) \geq 1-\alpha$ and $P_a(p_2) \leq \beta$

$$n \geq 1; c_1 \geq 0; c_2 > c_1 \qquad (7)$$

The design parameters such as n, c_1 and c_2 are determined for various values of p_1, p_2, α, β and m. The optimal parameters of the RGS plan under gamma-Poisson model is tabulated in Tables 1-5 for the shape parameters m=5, 10, 25, 50 and 100 respectively. It should be noted that the parametric values m in the prior distribution, range over the interval (0, ∞). It is also observed from the tables that the probability of acceptance of gamma-Poisson RGS plan do not differ much from those of conventional Poisson RGS plans for each value of p for larger values of m; that is, the OC function of gamma-Poisson RGS plan converges to Poisson RGS plan.

<Tables 1-5 are around here>

In this paper, we have assumed that the shape parameter of the gamma-Poisson distribution is known. The proposed plan can also be used for the situation where the shape parameter is unknown. Normally, producers keep the record of the estimated shape parameter value for their product or it can be estimated from the available data. From these tables we can observe interesting trends in the parameter values. For the same values of p_1, p_2, α, β, as the value of the shape parameter of the gamma-Poisson model increases, there exists a drastic reduction in the sample size. Also, when p_2 increases, the sample size decreases for other fixed values and at the same time, as p_1 increases, the sample size also increases.

3.1. Examples

Suppose that an experimenter wants to run an experiment to make a decision on a product, whether to accept or reject it and wants to implement gamma-Poisson RGS plan. Suppose that the AQL and LQL values are given as, p_1 = 1%, p_2 = 6%, with α = 5% and β = 10% and the estimated value of m=25. Under these requirements, from Table 3 one can find the values of optimum parameters as n=50, c_1=0 and c_2=2 with ASN=80.260. This plan is operated as follows:

Step (i) : Select a random sample 50 from the lot and observe the number of non-conforming items.

Step (ii) : If there is no nonconforming item, then accept the lot. If more than 2 non-conforming items are observed, then immediately reject the lot. If the number of non-conforming items is 1 or 2, then repeat the step (i) and (ii) until a decision is made.

4. ADVANTAGES OF GAMMA-POISSON RGS PLAN

In this section, we will make a comparative study on the results of the proposed plan with the gamma-Poisson single sampling plan of Vijayaraghavan et al. [11]. Any sampling plan with minimum ASN would always be preferable. Table 7 gives the ASN of the gamma-Poisson RGS plan and gamma-Poisson single sampling plan. Here we consider three values of the shape parameter, namely 5, 50 and 150 for some selected combinations of AQL and LQL values. From this table, it is easily observed that the ASN of the proposed plan is lesser than the ASN of the gamma-Poisson single sampling plan for all the combinations of the AQL and LQL.

<Table 6 is around here>

For further comparison, two figures of OC curves are presented. Figure 1 provides the OC curves of gamma-Poisson RGS plan along with the gamma Poisson single sampling plan and conventional Poisson RGS plans, all having same AQL=0.01 and LQL=0.06.

<Figure 1 is around here>

From this figure, it can be observed that the OC curve of gamma-Poisson RGS plan has desirable shape as a composite OC curve. For good quality, i.e. for smaller values of fraction nonconforming, the OC curve of the gamma-Poisson RGS

able 1. Optimal Gamma-Poisson RGS Plan for given p_1, p_2, α=5% and β=10% for m=5

p_1	p_2					
	0.05	0.06	0.07	0.08	0.09	0.10
0.005	72; 0,2 (104.902)	54; 0,1 (64.419)	46; 0,1 (54.965)	41; 0,1 (48.754)	36; 0,1 (42.946)	33; 0,1 (39.179)
0.010	79; 0,3 (141.781)	66; 0,3 (118.204)	51; 0,2 (74.644)	45; 0,2 (65.563)	40; 0,2 (58.279)	36; 0,2 (52.451)
0.015	94; 0,5 (250.050)	72; 0,4 (158.139)	56; 0,3 (101.136)	49; 0,3 (88.494)	44; 0,3 (78.803)	36; 0,2 (52.451)
0.020	115; 0,8 (528.107)	84; 0,6 (271.355)	67; 0,5 (178.738)	54; 0,4 (118.604)	44; 0,3 (78.803)	40; 0,3 (71.051)
0.025	150; 0,13 (1436.667)	96; 0,8 (439.296)	72; 0,6 (232.589)	59; 0,5 (156.054)	48; 0,4 (105.426)	43; 0,4 (94.923)

Table 2. Optimal Gamma-Poisson RGS Plan for given p_1, p_2, α=5% and β =10% for m =10

p_1	p_2					
	0.05	0.06	0.07	0.08	0.09	0.10
0.005	57; 0,1 (69.570)	48; 0,1 (58.392)	41; 0,1 (49.931)	36; 0,1 (43.793)	32; 0,1 (38.927)	29; 0,1 (35.201)
0.010	71; 0,3 (140.913)	53; 0,2 (82.163)	46; 0,2 (70.716)	40; 0,2 (61.749)	36; 0,2 (55.114)	29; 0,1 (35.201)
0.015	78; 0,4 (200.035)	59; 0,3 (117.439)	51; 0,3 (100.634)	40; 0,2 (61.749)	36; 0,2 (55.114)	32; 0,2 (49.399)
0.020	92; 0,6 (390.834)	71; 0,5 (233.99)	56; 0,4 (142.598)	44; 0,3 (88.096)	39; 0,3 (78.315)	36; 0,3 (70.426)
0.025	113; 0,9 (969.599)	77; 0,6 (324.175)	61; 0,5 (200.215)	49; 0,4 (124.773)	43; 0,4 (111.467)	36; 0,3 (70.425)

Table 3. Optimal Gamma-Poisson RGS Plan for given p_1, p_2, α =5% and β =10% for m =25

p_1	p_2					
	0.05	0.06	0.07	0.08	0.09	0.10
0.005	53; 0,1 (65.677)	45; 0,1 (55.394)	38; 0,1 (47.026)	34; 0,1 (41.744)	30; 0,1 (36.930)	27; 0,1 (33.237)
0.010	60; 0,2 (96.312)	50; 0,2 (80.260)	43; 0,2 (68.848)	37; 0,2 (60.007)	30; 0,1 (36.930)	27; 0,1 (33.237)
0.015	73; 0,4 (214.455)	55; 0,3 (119.441)	47; 0,3 (102.439)	37; 0,2 (60.007)	33; 0,2 (53.381)	30; 0,2 (48.156)
0.020	80; 0,5 (319.717)	61; 0,4 (178.404)	47; 0,3 (102.439)	42; 0,3 (89.264)	37; 0,3 (79.486)	30; 0,2 (48.156)
0.025	94; 0,7 (695.623)	67; 0,5 (265.022)	52; 0,4 (153.447)	46; 0,4 (133.343)	37; 0,3 (79.486)	33; 0,3 (71.665)
0.03	123; 0,11 (2866.564)	79; 0,7 (570.358)	63; 0,6 (331.378)	50; 0,5 (199.823)	41; 0,4 (118.324)	37; 0,4 (106.309)
0.035	164; 0,17 (19610.98)	97; 0,10 (1673.085)	68; 0,7 (484.932)	55; 0,6 (290.946)	45; 0,5 (175.284)	37; 0,4 (106.309)
0.04	269; 0,33 (1214995.0)	126; 0,15 (8634.507)	83; 0,10 (1441.903)	59; 0,7 (431.253)	49; 0,6 (257.739)	40; 0,5 (159.858)
0.045	392; 0,53 (58352120.0)	171; 0,23 (84844.75)	98; 0,13 (3949.326)	68; 0,9 (894.419)	53; 0,7 (375.645)	44; 0,6 (232.757)
0.05	***	277; 0,43 (7796767.0)	122; 0,18 (18799.23)	81; 0,12 (2537.945)	61; 0,9 (775.205)	47; 0,7 (347.812)

*** Plan does not exist (Value given in bracket is the ASN of the plan at LQL)

Table 4. Optimal Gamma-Poisson RGS Plan for given p_1, p_2, α =5% and β =10% for m =50

p_1	p_2					
	0.05	*0.06*	*0.07*	*0.08*	*0.09*	*0.10*
0.005	52; 0,1 (64.674)	44; 0,1 (54.415)	37; 0,1 (46.083)	33; 0,1 (40.812)	29; 0,1 (36.017)	26; 0,1 (32.337)
0.010	58; 0,2 (95.420)	49; 0,2 (79.732)	42; 0,2 (68.342)	37; 0,2 (59.879)	29; 0,1 (36.017)	26; 0,1 (32.337)
0.015	65; 0,3 (144.537)	54; 0,3 (120.544)	42; 0,2 (68.342)	37; 0,2 (59.879)	33; 0,2 (53.262)	29; 0,2 (47.710)
0.020	79; 0,5 (335.589)	60; 0,4 (183.480)	46; 0,3 (103.489)	41; 0,3 (90.121)	36; 0,3 (80.363)	29; 0,2 (47.710)
0.025	93; 0,7 (772.423)	66; 0,5 (278.814)	51; 0,4 (158.225)	45; 0,4 (137.610)	36; 0,3 (80.363)	33; 0,3 (71.982)
0.030	115; 0,10 (2510.404)	78; 0,7 (634.935)	56; 0,5 (241.892)	45; 0,4 (137.610)	40; 0,4 (122.320)	33; 0,3 (71.982)
0.035	151; 0,15 (15803.8)	90; 0,9 (1409.954)	67; 0,7 (541.748)	49; 0,5 (211.656)	44; 0,5 (185.876)	36; 0,4 (110.088)

0.04	227; 0,26 (609736.1)	114; 0,13 (6394.118)	77; 0,9 (1215.395)	58; 0,7 (484.974)	48; 0,6 (281.283)	40; 0,5 (165.280)
0.045	323; 0,40 (27787200)	155; 0,20 (73611.320)	92; 0,12 (3900.290)	67; 0,9 (1081.714)	52; 0,7 (423.829)	43; 0,6 (255.116)
0.050	***	223; 0,32 (3104786.0)	113; 0,16 (15982.23)	76; 0,11 (2343.716)	56; 0,8 (632.927)	47; 0,7 (377.497)

Table 5. Optimal Gamma-Poisson RGS Plan for given p_1, p_2, α =5% and β =10% for m =100

p_1	p_2					
	0.05	0.06	0.07	0.08	0.09	0.10
0.005	52; 0,1 (64.562)	43; 0,1 (53.544)	37; 0,1 (46.005)	32; 0,1 (39.964)	29; 0,1 (35.954)	26; 0,1 (32.281)
0.010	58; 0,2 (95.332)	48; 0,2 (79.345)	41; 0,2 (67.967)	36; 0,2 (59.509)	29; 0,1 (35.954)	26; 0,1 (32.281)
0.015	64; 0,3 (145.579)	54; 0,3 (120.876)	41; 0,2 (67.967)	36; 0,2 (59.509)	32; 0,2 (52.897)	29; 0,2 (47.666)
0.020	78; 0,5 (347.599)	59; 0,4 (187.624)	46; 0,3 (103.796)	40; 0,3 (90.987)	32; 0,2 (52.897)	29; 0,2 (47.666)
0.025	93; 0,7 (810.018)	65; 0,5 (289.665)	51; 0,4 (159.774)	45; 0,4 (138.893)	36; 0,3 (80.584)	32; 0,3 (72.789)
0.030	107; 0,9 (1899.186)	77; 0,7 (684.896)	56; 0,5 (246.686)	45; 0,4 (138.893)	40; 0,4 (123.460)	32; 0,3 (72.789)
0.035	144; 0,14 (13773.34)	89; 0,9 (1592.238)	66; 0,7 (587.054)	49; 0,5 (215.850)	44; 0,5 (189.410)	36; 0,4 (111.114)
0.04	216; 0,24 (553953.1)	114; 0,13 (7748.914)	71; 0,8 (902.124)	58; 0,7 (508.715)	48; 0,6 (290.236)	39; 0,5 (173.799)
0.045	284; 0,34 (162222280.0)	144; 0,18 (52844.85)	87; 0,11 (3047.631)	63; 0,8 (759.673)	52; 0,7 (443.519)	43; 0,6 (263.362)
0.050	***	211; 0,29 (2435293.0)	108; 0,15 (14508.74)	76; 0,11 (2685.128)	56; 0,8 (675.265)	47; 0,7 (395.311)

*** Plan does not exist (Value given in bracket is the ASN of the plan at LQL)

Table 6. Average Sample Number of the Proposed Plan and Gamma-Poisson Single Sampling Plan for Specified p_1, p_2, α =5% and β=10%

p_1	p_2	ASN at p_2					
		m=5		m=50		m=150	
		SSP	RGS	SSP	RGS	SSP	RGS
0.005	0.05	191	104.902	111	64.674	108	63.751
0.005	0.06	124	64.419	67	54.415	66	53.513 79.321
0.01	0.06	264	118.204	116	79.732	113	67.949
0.01	0.07	166	74.644	79	68.342	77	67.949
0.015	0.07	492	101.136	119	68.342	116	59.491
0.015	0.08	276	88.494	87	59.879	85	91.083
0.02	0.08	1433	118.604	121	90.121	118	52.881
0.02	0.09	451	71.051	93	80.363	91	80.656
0.025	0.09	***	105.426	123	80.363	119	72.866
0.025	0.10	***	94.923	97	71.982	95	

*** Plan does not exist (Value given in bracket is the ASN of the plan at LQL)

Table 7 : Lot Acceptance Probabilities of the Proposed Plan at AQL and LQL for Different Shape Parameters when m_0=10.0

p_1	p_2	m =9.5		m=10.0		m =10.5		m =11.0	
		$P_a(p_1)$	$P_a(p_2)$	$P_a(p_1)$	$P_a(p_2)$	$P_a(p_1)$	$P_a(p_2)$	$P_a(p_1)$	$P_a(p_2)$
0.005	0.05	0.95465	0.10102	0.95477	0.09943	0.95488	0.09799	0.95498	0.09667
0.005	0.06	0.96770	0.09839	0.96780	0.09682	0.96788	0.09539	0.96796	0.09408
0.01	0.06	0.96747	0.09986	0.96770	0.09801	0.96791	0.09632	0.96810	0.09480
0.01	0.07	0.97854	0.09612	0.97871	0.09429	0.97886	0.09263	0.97901	0.09112
0.015	0.07	0.97758	0.09523	0.97787	0.09319	0.97813	0.09135	0.97838	0.08968
0.015	0.08	0.95337	0.09797	0.95366	0.09612	0.95393	0.09446	0.95417	0.09294
0.02	0.08	0.96181	0.10019	0.96224	0.09812	0.96263	0.09630	0.96298	0.09453
0.02	0.09	0.97585	0.10122	0.97616	0.09913	0.97644	0.09725	0.97669	0.09553
0.025	0.09	0.97749	0.10063	0.97790	0.09838	0.97827	0.09635	0.97861	0.09450
0.025	0.10	0.95845	0.09239	0.95890	0.09037	0.95931	0.08855	0.95968	0.08690

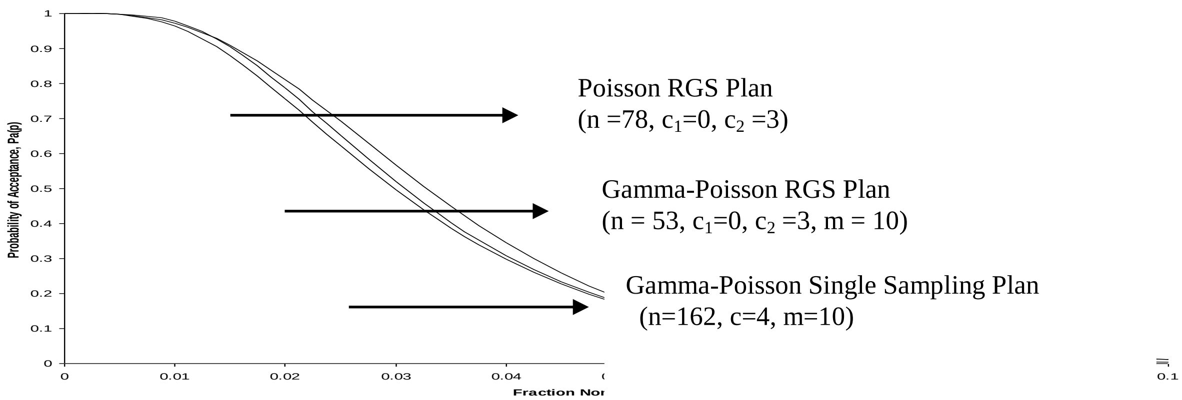

Figure 1. Operating Characteristic Curves of Poisson RGS Plan, Gamma-Poisson Single Sampling Plan and Gamma Poisson RGS Plans Corresponding to p_1=0.01 (α=0.05) and p_2=0.06 (β=0.10)

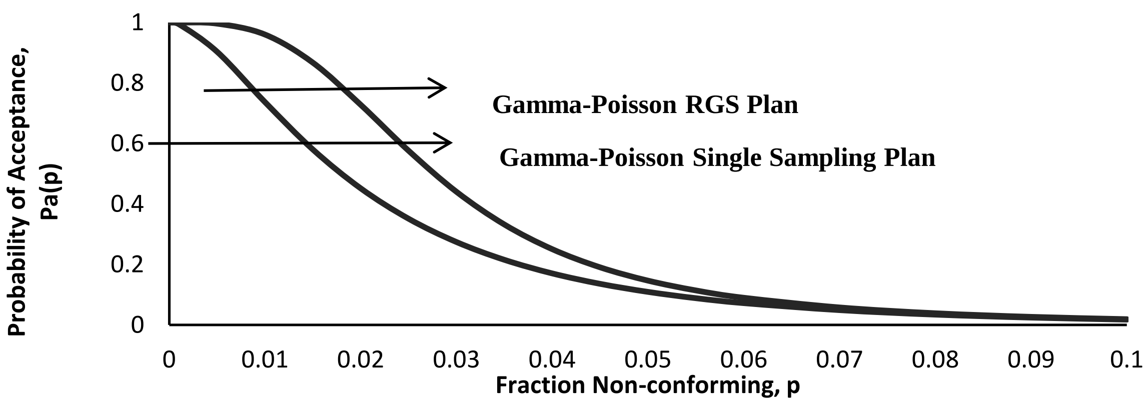

Figure 2. Operating Characteristic Curves of Gamma-Poisson Single Sampling Plan and Gamma Poisson RGS Plan with Same Parametric Values

plan coincides with the OC curve of the conventional Poisson RGS plan. As quality deteriorates the OC curve of the gamma-Poisson RGS plan moves toward that for the gamma-Poisson single sampling plan and comes close to it beyond the indifference quality level. It indicates that all the gamma-Poisson RGS plans, in general, protect the producer's interest against good quality levels and at the same time safeguard the consumer's interest against poor quality levels. Figure 2 gives the OC curves of single sampling plan RGS plan under gamma-Poisson distribution for fixed parameters. Here we consider the gamma-Poisson single sampling plan with parameters (n=100, c=1, m=5) and gamma-Poisson RGS plan with parameters (n=100, c_1=1, c_2=3, m=5). This figure shows that the gamma-Poisson RGS plan provides additional protection to the producer from the risk of rejecting the lots of good quality compared to the gamma-Poisson single sampling plan. For example, when the lot fraction non-conforming is 0.005, under the gamma-Poisson single sampling plan, the producer has 9.68% (probability of acceptance at p=0.01 is 0.9032) risk of getting the lot rejected ,whereas under the gamma-Poisson RGS plan, the producer will meet only 0.39% (P_a=0.9961) of risk. That is for good quality levels, the gamma-Poisson RGS plan will give themore probability of acceptance. When the quality deteriorates, the OC curve of gamma-Poisson RGS plan converges with the OC curve of the gamma-Poisson single sampling plan. For instance, when the lot fraction non-conforming is 0.07, both plans will have almost 5% probability of acceptance. Hence it is observed that gamma-Poisson RGS plan has more probability of acceptance than the gamma-Poisson single sampling plan when the lot quality is good and at the same time safe-guarding the consumers.

<Figure 2 is around here>

5. EFFECTS OF MISSPECIFICATION OF SHAPE PARAMETER

Misspecification of shape parameters of the distribution is taking a major role in the statistical quality control and distributions theory. Since the shape parameter is assumed to be known, one may be interested in the misspecification of this parameter. The effects of misspecification of the parameters have been studied by many authors see for example Keats et al. [20]. In this section, we would like to study the effect of misspecification of the shape parameter on the probability of acceptance of the lots of the proposed sampling plan under the Gamma-Poisson model. Suppose that m be the specified shape parameter and m_0 be the true shape parameter. The probability of acceptance for the proposed plan for the specified shape parameters at both AQL and LQL is calculated and tabulated in Table 8 for some selected combinations of p_1 and p_2 when α =5% and β =10%. In this table, we consider the true shape parameter is m_0=10. So the parameter values of the proposed plan are taken from Table 2. From Table 7, it is observed that when the specified shape parameter is less than the true shape parameter, then the probability of acceptance of the proposed plan at AQL is decreasing and at the same time the probability of acceptance is increasing at LQL for any combinations of p_1, p_2. However, if the specified shape parameter is more than the true shape parameter then the probability of acceptance at AQL is increasing and at LQL it is decreasing. The same trend can be observed for all combinations of AQL and LQL. It indicates that the higher value of the specified shape parameter will reduce the sum of producer and consumer risks. Therefore, the higher value of the shape parameter would be safer to be specified when the exact value is not known.

<Table 7 is around here>

6. CONCLUDING REMARKS

In this paper, we have developed a Bayesian RGS plan under the gamma-Poisson distribution. The optimal design parameters of the proposed plan are determined using the two points on the OC curve approach. The proposed plan is better than the single sampling plan for the gamma-Poisson distribution in terms of minimum ASN. The proposed plan provides the lesser ASN than the existing sampling plans. So, the RGS plan based on the gamma-Poisson model performs better than the conventional single, RGS plan and the gamma-Poisson single sampling plan and the proposed plan can be easily applied for the industrial use.

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